Using ideas of 'resource system' to analyse the work of mathematics teaching

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Basic ideas of ‘resource’ and ‘system’

- In established everyday usage, a resource is an asset – typically a monetary, material or human asset– that is capable of providing some form of support.
- The central idea of a 'system' is one of organisation:
  - Some structure resulting from multiple entities being organised to form a functioning whole;
  - Some scheme or method which provides a basis for this type of organisation.
- Traditionally education has made use of two ideas/types of 'resource system’ to support teaching and learning:
  - The textbook – a systematic curriculum scheme combining many units of resource into a coherent programme;
  - The library – a repository of resources organised systematically to make its contents readily searchable and usable.

Euclid's *Elements* as a systematic logical organisation of resources

- The *Elements* was a crucial text in the development of Western mathematics.
- It was created by Euclid around 300 BCE and widely studied until the early years of the twentieth century.
- Thanks to the remarkable Library of Alexandria, Euclid was able to draw on disparate mathematical texts from across the ancient world in compiling the *Elements*.
- What makes the *Elements* so remarkable?
- First, Euclid combined and adapted mathematical material from an extensive range of existing (re)sources to produce a comprehensive text presenting a core of classical mathematics.
- Foremost, however, it established a systematic approach to organising this material in a logically coherent way.

Global and local structure in the *Elements*

- A global structure for the text is established as follows:
  - It starts with 'definitions’ of entities and axioms, taken to be self-evident, either in the form of 'postulates' about geometrical entities or 'common notions' about magnitudes.
  - From these, it derives a succession of 'propositions’ which are numbered sequentially and grouped thematically into 'books’.
- A consistent local template is used to treat each proposition:
  - An 'enunciation' states the situation given and the result sought.
  - If required, a 'setting-out’ provides a labelled figure exemplifying the situation; and a 'construction' or 'mechanism' elaborates the figure to support reasoning to produce the desired result.
  - A step-by-step 'proof’ shows the warrant for each step by indexing the relevant definition, axiom or prior proposition.
  - A 'conclusion' relates the demonstration back to the original enunciation.

Euclid's *Elements* as a text for study

- As a text for study, the *Elements* provides access not just to mathematical content but to the logical method employed to organise it.
- In the field of mathematics, the *Elements* came to fulfil an important sociocognitive function, establishing a shared argumentative framework for the diffusion of knowledge.
- In the field of education, the *Elements* was embraced by an influential approach which sought to expose students to the classical models of thought displayed in 'great books'.
- Indeed, the *Elements* came to be studied primarily for the habits of mind that such study was thought to inculcate.
- Induction to the Euclidean system through 'exposure to the *Elements*' was intended to teach students to reason in an abstract realm removed from sensory perception.

From ‘great book’ to ‘school book’

- Yet actual educational practice could be very different.
- For example, in England in the early 20th century, reformers criticised one of the requirements to graduate at Oxford University: to memorise two books of Euclid, even down to the lettering of figures, with no original exercises required.
- Such examples show how the *Elements* had often become associated with a reductive mnemonic pedagogy.
- As reformers gained the upper hand, then, the 'great book' gave way to the 'school book’.
- Texts written specifically to introduce school pupils to geometry gave a place to practical experimentation and took a less restrictive approach to modes of reasoning – in line with the didactical precepts of the reform movement.

Durell's *A New Geometry* as a systematic didactical organisation of resources

- To characterise the 'school books' which took the educational place of Euclid's 'great book', I will use the example of Durell's *A New Geometry for Schools*.
- In his review of English mathematics textbooks of the 20th century, Quadling describes Durell as "the most prolific author of the century” so that his "name was for many pupils almost synonymous with mathematics”.
- In the preface to the text, Durell sets out his didactically inspired organisational scheme for the systematic sequencing of activity within each topical unit of the text.
- This scheme involves four stages, with each stage linked to particular types of learning goal and a corresponding form of classroom activity.

Durell’s 4-stage didactical scheme

- The earlier stages provide an informal introduction:
  - Stage (i) *Examples for oral discussion to* give pupils a clear understanding of facts, familiarise them with arguments used later in formal proofs, and train them in methods for solving related problems (‘riders’).
  - Stage (ii) *An exercise of numerical examples* to give practice in applying facts and ensure a firm grasp of them.
- The later stages develop a more formal treatment:
  - Stage (iii) *Formal proofs of the corresponding theorems,* focusing on the key theorem in each group, to give practice in writing out proofs.
  - Stage (iv) *An exercise of riders,* in which early examples are simple applications and later examples include establishing related theorems in the group.

Durell's *Geometry* as a text for study

- Consistent use of this scheme throughout the textbook accustoms teacher and pupils to the staged approach, allowing them to focus on the mathematical tasks and learning goals in play.
- The third stage provides a degree of continuity with the approach of the *Elements*, but is more economical in the theorems chosen to receive formal proof.
- Organising material conceptually around a key theorem incorporates a powerful learning principle.
- Equally the two final stages produce a balance between exposure to formal proof and experience of solving riders (including those relating to other theorems in the group).

Durell's *A New Geometry* as a systematic didactical organisation of resources

- The core of *A New Geometry* is the sequence of topic-specific resource units forming the individual chapters of the book, each employing the staged organisation just outlined.
- This corresponds to the first sense of resource system as a systematic curriculum sequence combining resources into a coherent programme.
- Ancillary features of the text − indexes and appendices - correspond to the second sense of resource system as a repository of readily searchable and usable resources.
- Such a text is designed as a compact, comprehensive *all-in-one* resource system, meeting the various needs of teachers and pupils over the course as a whole.
- It is this explicit and systematic didactical organisation which makes Durell's text identifiably a textbook.

Beyond the ‘teacher-proof’ text

- We have seen how both these texts were designed so as to provide a particular type of organised resource system.
- But we have also seen how the users of such a text may fail to honour the organising system, as illustrated by the sometimes debased treatment of the *Elements*.
- Equally, amongst more than 1000 editions of the *Elements,* many sought to ‘improve’ its organising system, often with didactical and pedagogical considerations in mind.
- Indeed, beyond a certain point, while still clearly indebted to the *Elements*, reformed texts made such fundamental changes to the organising system as to claim distinct identities, as Durell’s *Geometry* illustrates.
- Equally, recognising that teachers often seek a text aligned with their preferred teaching approach, Durell produced several other geometry texts, organised in alternative ways.

Summary of part 1 – the comprehensive resource system

- In everyday language, a 'resource' is some form of supportive asset.
- Likewise a 'system' is some form of methodical organisation.
- Viewed in such terms, texts such as Euclid's *Elements* (or *The Nine Chapters on the Mathematical Art* 九章算术) can be seen as a form of mathematical 'resource system'.
- These foundational mathematical texts adapted and organised units of material from many sources.
- Later their place was taken by school textbooks such as Durell's *Geometry*.
- Through an overarching didactical organisation, textbooks aim to provide a comprehensive curricular 'resource system' adapted to the various needs of teachers and students.

The genesis of the curricular 'resource'

- In the field of education, a specialised idea of 'resource' came into use during the 1960s, referring specifically to materials supporting curricular activity.
- The emergence of this era of curricular 'resources’ was fostered by technological changes – notably the increasing availability of audio-visual and reprographic facilities.
- These changes broadened the range of media in which curricular materials could be created, and facilitated their local production and reproduction.
- More recently further technological changes have enabled a shift towards digitalising resources and accessing them online.
- This shift has facilitated the exchange of curricular materials on a much wider scale, accompanied by the development of organisations aiming to foster such exchanges.

‘Resource-based’ learning and teaching

- In Western countries, there has been increasing interest in 'resource-based' forms of learning and teaching, involving, respectively, more independent study by pupils and more localised curriculum design by teachers.
- The institutionalisation of such trends has been marked by the renaming of the traditional library as the modern 'resource centre’.
- As such approaches have become increasingly influential, there has been a shift away from using a comprehensive course textbook towards combining many smaller resource units from different sources.
- Nevertheless, some skeleton 'resource system’ remains necessary, providing a framework for organising resources so as to provide a comprehensive and coherent curriculum.

SMILE as a systematic curricular organisation of multi-sourced resources

- A key feature of early resource-based initiatives in school mathematics, such as SMILE in England, was development of a curriculum map into which carefully chosen (or specially devised) 'tasks' from different sources could be inserted.
- In SMILE, the basis for this map was a grid combining the dimensions of topic area and of conceptual level to create a system of cells into which each task resource was mapped.
- Within this grid, further dependencies between cells and between tasks could then be recorded in greater detail.
- Teacher and student used this grid to plan, taking account of the attainment of the student and tasks already undertaken.
- Guided by the grid, they chose appropriate tasks for the student to undertake to support progression in learning.

GAIM as a systematic curricular and cognitive organisation of assessment tasks

- This curriculum map came to be paralleled by a graded assessment system, GAIM.
- GAIM created a levelled system of criteria describing specific cognitively-based strategies which represent significant steps in mathematical development.
- At this time, what was particularly notable about this development was the creation of a system which combined a curricular model of domains of mathematical knowledge with a cognitive model of progression in mathematical thinking.

Math-Mapper as a systematic developmental organisation of curriculum/assessment tasks

- A modern counterpart of this type of subject-specific 'shell' – for managing curricular resources and integrating assessment – is the Math-Mapper digital learning system organised around learning trajectories.
- To guide the learning efforts of students (and teacher support for these efforts) it sets appropriate learning targets and identifies corresponding learning opportunities (through digital curriculum resources mapped into the 'shell').
- Then it makes diagnostic assessments to provide feedback on the success of these efforts, and analyses progress to inform appropriate next steps in instruction and learning.

From a multi-sourced collection of resources to an organised system

- The local insertion of resources into such 'shells' makes considerable demands on teachers.
- Thus there are versions of both SMILE and Math-Mapper prepopulated with suitable curricular resources, so taking on a form closer to the traditional textbook or modern digital curriculum programme.
- Indeed SMILE developed into a comprehensive curriculum programme, distributed well beyond the contributing schools and teachers, and sustained by a group of core participants responsible for ‘minding the system’.
- Such trends indicate the continuing importance of coherent, comprehensive, externally developed systems of resources in supporting mathematics education in schools.

Summary of part 2: the multi-sourced resource system

- In Western countries, during the 1960s, a specialised educational idea of 'resource' emerged, referring to any type of material capable of supporting curricular activity.
- Interest has subsequently grown in 'resource-based' forms of learning and teaching.
- These involve, respectively, more independent study by pupils and more localised curriculum design by teachers.
- Often, too, this involves a shift away from using a single course textbook towards combining many smaller resource units taken from different sources.
- Nevertheless, some kind of skeleton 'resource system’ is still needed to organise many disparate resources so as to provide a comprehensive and coherent curriculum.

Overview of part 3 – recent ideas from research

- Such developments have led researchers to give renewed attention to teachers’ use of curricular resources.
- This has produced further ideas of 'resource system' attuned to the particular focus of research.
- Ruthven (2009) focuses on the practices and rationales through which mathematics teachers seek to create a functional 'resource system’, particularly one incorporating digital resources alongside (or in place of) classical ones.
- Gueudet & Trouche (2009) focus on the organisation and evolution of the 'resource system' consisting of the structured collection of resources used by a teacher in planning and delivering lessons, and on the schemes guiding these processes.

‘Resource’ and ‘system’ in Ruthven (2009)

- For Ruthven (2009), ‘resources’ comprise the mathematical tools and curriculum materials in use, and the ‘system’ relates to the organisation of their modes of use.
- The focus is on the functionality of the resource system, particularly inasmuch as it incorporates digital resources alongside (or in place of) classical resources.
- The practices and rationales reported by teachers provide the evidential base on which the functioning of the resource system (as perceived by them) is reconstructed.

The ‘resource system’ for a geometry unit

- As an example, consider how the resource system used by a teacher for an existing geometry unit for a first-year secondary-school class was evolving.
- As in the past, the unit started with paper-and-pencil work on constructions with classical tools (straight edge/compass).
- It had now been extended to include a new computer-based investigation, making use of dynamic geometry software.
- The resource system for this curriculum unit, then, was a dual one, involving sequential use of classical then digital tools.
- The analysis examines the practices and rationales through which the teacher sought to make this resource system functional.

Clarifying how digital tools complement classical tools

- The teacher noted how the software supported exploration of different cases and overcame practical difficulties of students attempting such an investigation by hand:
  - *You can move it around and see that it's always the case and not just that one off example. But I also think they get bogged down with the technicalities of drawing the things and getting their compasses right, and [dealing with] their pencils broken.*
- But the teacher saw the contribution of the software as going beyond ease and accuracy; using it required properties to be formulated precisely in geometrical terms:
  - *And it's the precision of realising that the compass construction... is about the definition of what the perpendicular bisector is... And Geometer's Sketchpad forces you to use the geometry and know the actual properties that you can explore.*

Reconciling incongruence of tools as distinctive functions

- At the same time, the teacher felt that there was a degree of incongruence between tools because certain classical techniques appeared to lack digital counterparts.
    - *When you do compasses, you use circles and arcs, and you keep your compasses the same. And I say to them: "Never move your compasses once you've started drawing."... Well Geometer's Sketchpad doesn't use that notion at all.*
- Accordingly, digital and classical were seen as involving different methods and having distinct functions:
    - *So it's a different method. Whether they then can translate that into compasses and pencil construction, I don't think there's a great deal of connection. I don't think it's a way of teaching constructions, it's a way of exploring the geometry.*

Justifying familiarising students with software operation

- The teacher was willing to invest time in familiarising students with using the software because he saw this as engaging them in disciplined interaction with a geometric system:
  - *I always introduce Geometer's Sketchpad by saying "It's a very specific, you've got to tell it. It's not just drawing, it's drawing using mathematical rules."*
- This also capitalised on earlier investment in using classical tools, an economy of the joint resource system:
  - *And also they're doing the constructions by hand first, to see, getting all the words, the key words, out of the way.*
- Likewise, he was willing to spend time making students aware of the underlying construction process of figures:
  - *I always like to start with a blank page and actually put it together in front of the students so they can see where it's coming from.*

Developing student use of distinctive digital techniques

- The teacher supported students experiencing difficulties in manipulating the software, and turned these difficulties to advantage in reinforcing the mathematical issue at hand:
  - *Understanding the idea of perpendicular bisector... you select the line and the [mid]point... There's a few people that missed that and drew random lines... So you have to click away and de-select things, and that caused a bit of confusion… But... quite a few discussions I had with them emphasised which line is perpendicular to that edge.*
- Likewise, he prompted students to use the most distinctive functionality of the software, dragging a dynamic figure:
  - *They didn't spot that they all met at a point as easily... I don't think anybody got that without some sort of prompting. It's not that they didn't notice it, but they didn't see it as a significant thing to look for... When I talked about meeting at a point, they were able to move it around.*

Establishing norms for appropriate digital tool use

- Some features of the software were not wholly welcome.
- For example, students could be deflected from a mathematical focus by an overconcern with presentation.
  - *They spend… three quarters of the lesson making the font look nice and making it all look pretty [but] getting away from the maths.*
- The teacher had tried out a new technique for managing this, by projecting an example to illustrate appropriate use:
  - *It showed that I did want them to think about the presentation, I did want them to slightly adjust the font and change the colours a little bit, to emphasise the maths, not to make it just look pretty.*
- Here the teacher is establishing sociomathematical norms for using the tool to create a better functioning resource system.

Replacing old tools and techniques by better new ones

- The teacher was developing ideas about how using digital text-boxes could make students more willing to consider revising (and so rethinking) their written observations:
  - *They had to write in [a text box], and they could change it and edit it. They could… then think about what they were writing, how they describe… With handwritten, if someone writes a whole sentence next to a neat diagram, and you say, "What about that word? Can you add this in?", you've just ruined their work. But with technology you can just change it… add on an extra bit, and they don't mind.*
- This was helping him to develop students' capacity to express themselves clearly in geometrical terms:
  - *I was focusing on getting them to write a rule clearly… There were a lot writing "They all meet"... So we were trying to discuss what "all" meant, and a girl at the back had "The perpendicular bisectors meet"…. "Meet at a point": having that sort of sentence there.*

‘Resource’ and ‘system’ in Gueudet & Trouche (2009)

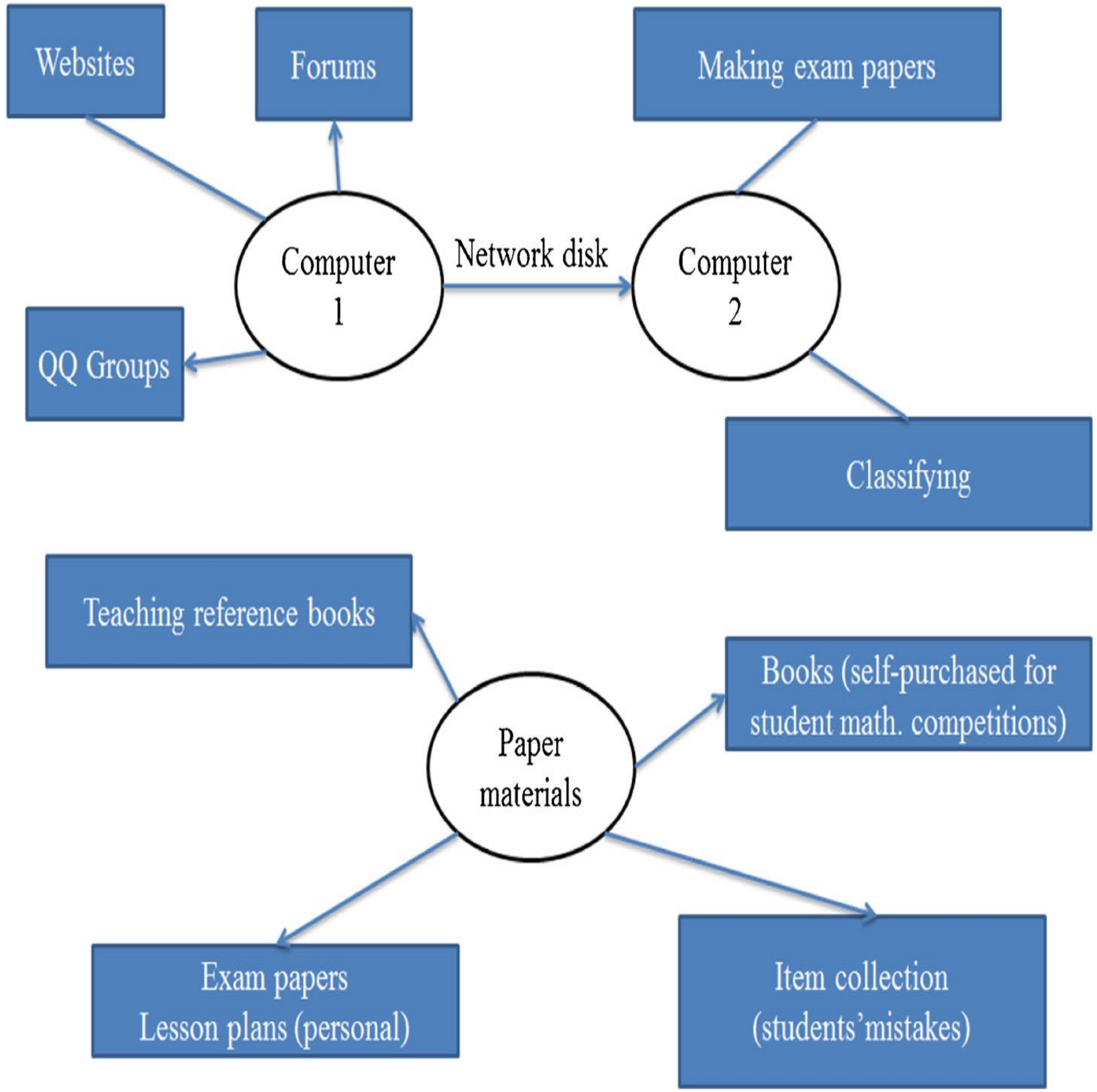
- For Gueudet & Trouche (2009), ‘resources’ comprise not only curricular materials but any other sources on which teachers draw in planning and conducting lessons.
- Thus their ‘resources’ can be non-material, such as discussions between teachers or student reactions to lessons.
- Users convert such resources into tools by constructing cognitive ‘utilization schemes’: the combination of resource and scheme is termed a ‘document’.
- The focus in this approach is on the schematic structures which organise teachers’ ‘documentation systems’.
- In later work, however, Gueudet & Trouche do use ‘resource system’ to refer to the set of resources within a teacher’s documentation system (i.e. without the schemes).

The ‘resource system’ of an expert teacher

- A central hypothesis of this approach is that teachers’ resource systems reflect their working practices and professional expertise, and co-evolve with these, and so can provide a window into them.
- This approach was taken in a research study (Pepin, Xu, Trouche & Wang, 2017) of the resource systems of Chinese mathematics teachers, regarded as experts by the education authorities, leading teacher research groups in their school.
- Here we will focus on one teacher’s resource system.
- As a preliminary to interviewing the teacher about his use of resources, the researchers asked him to draw a map of his resources with respect to the different associated activities.

The teacher's Schematic Representation of his Resource System (SRRS)

- The basic parts of the resource system are:
- home computer (for accessing and storing resources from online groups, forums and sites);
- office computer (for preparing lessons and tests);
- paper material (for external resources, and own listings of student errors).



Exploitation and evolution of the teacher's resource system

- For lesson preparation the teacher reported that he would first develop a basic lesson plan by himself, knowing what the curriculum standards were.
- He would then turn to other resources such as teaching guidelines, curriculum standards and selected teaching-aid documents.
- Here his first source was the "lesson preparation group" (the grade-specific part of the teacher research group); and his second source, mathematics resource websites.
- In post-lesson review, he would write down his reflections about students, their thinking, their interactions, and so on.
- The teacher was proud that he had been good at collecting and organizing his resources over the years, and had modified and appropriated them to become his own.

A ‘utilization scheme’ composed of ‘rules of action’ and ‘operational invariants’

- Perhaps the main importance of highlighting the cognitive aspect is to analyse the ‘utilization schemes’ that enable ‘resource sets’ to form ‘documentation systems’.
- Although this aspect is not prominent in many of the studies espousing the ‘documentational approach’, an example of a study where it is is Gueudet (2017).
- The table on the next slide summarises an analysis, from that study, of schemes associated with a university mathematics teacher’s use of resources.
- In particular, it identifies the ‘rules of action’ (forms of use) and ‘operational invariants’ (underlying organising principles) comprising the ‘utilization schemes’ associated with the use of particular resources for particular purposes.

Analysis of schemes associated with one university mathematics teacher's use of resources.

Aims	Resources used	Rules of actions (Way to use the resources)	Operational invariants (Reasons for using them this way)
Preparing the first lecture on numerical methods	Curriculum of year 1 and year 2 semester 1 Written notes (his notes and the notes of the previous teacher)	The first lecture starts by recalling previous knowledge which will be used The first lecture also introduces what is new in the perspective of the course	The new lecture must be connected with the students' previous knowledge. "The students must learn that the results they know are connected: mean value theorem, Taylor formula, etc.".
Preparing and setting up exercises for a tutorial on interpolation	Book (Crouzeix and Mignot 1984) Written notes (his notes) Previous exam texts Scilab Tables of values of functions, including an old table of a Bessel function	Some exercises are chosen from the book and adapted to the students' level Some exercises are chosen from the notes of the previous year Choose tables of values of functions and values which are not in the table.	The students must know how to use interpolation to produce approximate values of a function "Using links with the history of mathematics increases students' motivation"

Concluding thoughts: the protean ‘resource system’

- Ideas of 'resource system' differ considerably in the ways in which they demarcate 'resources' and formulate 'system'.
- Equally, closer examination shows that different perspectives situate 'resource system' in contrasting ways:
  - as adhering to a particular type of agent – teacher, student, designer – or as intervening between such agents;
  - as related to a specific educational entity – especially the text, the classroom, the course or the lesson – or as ranging across and beyond these;
  - as governing the structuring and/or exploitation of resources.
- Professionals and researchers have clearly found each of these variations useful for some purpose.
- Could we benefit from developing a correspondingly overarching notion of 'resource system’?

Key references

- **All slides**
  - Ruthven, K. (2019). The construct of 'resource system' as an analytic tool in understanding the work of teaching. In L. Trouche, G. Gueudet & B. Pepin (Eds.) *The 'Resource' Approach to Mathematics* Education (pp. 43-59). Springer.
- **Slides 3-6**
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- **Slides 7-10**
  - Durell, C. V. (1939). *A New Geometry for Schools*. Bell.
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Key references (continued)

- **Slides 21-28**

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  - Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: a study of the interpretative flexibility of educational software in classroom practice. *Computers & Education* 51(1), 297-317.

- **Slides 29-34**

  - Gueudet, G. (2017). University teachers' resources systems and documents. *International Journal of Research in Undergraduate Mathematics Education*, *3*(1), 198-224.
  - Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics* 71(3), 199–218.
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