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CURRENT PRACTICE IN USING DYNAMIC GEOMETRY TO TEACH ABOUT ANGLE PROPERTIES

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Recently, we have been examining how experienced teachers in well-regarded subject departments approach the use of ICT in secondary mathematics. Drawing on multiple sources, we identified 11 departments considered to be successful in terms of the quality both of the education that they provided, and of their integration of ICT into classroom practice. Next, we conducted focus group interviews with each of these departments, asking teachers to describe examples of successful practice in using ICT. Then, we invited teachers who had been articulate in support of particular types of practice to help us gain greater insight into these through lesson observations and post-lesson interviews. Dynamic geometry was nominated in around half the departments that we visited.

The way of using dynamic geometry systems (DGS) which teachers most commonly singled out involved establishing angle properties through dragging a geometric figure and displaying its angle measures. Typical topics mentioned were: vertically opposite and supplementary angles; corresponding and alternate angles; angle sums of the triangle and other polygons; and angle properties of the circle ('the circle theorems'). To investigate what teachers saw as important in making such use of dynamic geometry effective in their teaching, we attended lessons on these last three topics, observing and interviewing three teachers from two schools.

In the focus group at the first school, one teacher emphasised the efficiency of using DGS:

> I've used [named DGS] in the past, for circle theorems a lot… The one that I do like to do is the one with the circle theorem that says the angle at the centre is twice the angle at the circumference, because that covers the same theorem as the angles in the same segment and the angle at the semi-circle is ninety degrees, and you can cover a lot of different circle theorems by doing that one demonstration.

His colleague brought out the potential for students to play a more active part:

> I think it works even better if they can do it for themselves. I mean you have to guide them into what they are doing, but then… they are actually more or less discovering for themselves.

In the focus group at the other school, the third teacher that we went on to observe and interview reported using similarly structured investigations, but showed stronger concern with creating a sense of agency on the part of students:

> All of our angle work at [lower secondary level] is done on it… They work… [on] pre-designed… mini-investigations really… It gives them control over it… It's their work and

> they've found out how it works without us telling them. They've worked it out. So it's quite rewarding… Most of the tasks are not designed to move freely and openly; they're designed with what we want to achieve from it in mind. So, if we want them to see that the angles on a straight line add to one hundred and eighty, it's designed exactly for that purpose… So they should come to the right conclusion but… they feel that they've done it on their own and they've explained it.

Developing viable approaches to classroom DGS use

Important differences of basic approach to using DGS in lessons were related to teachers' views on the accessibility and value of DGS technique to their students.

The approach employed by the first teacher (teaching lower sets in Years 9 and 10) was to demonstrate to the class by projecting dynamic geometry figures from his laptop onto the ordinary classroom whiteboard. He doubted that getting students themselves to work with DGS would repay the time and effort required to develop the necessary technique:

> If I wanted the students to do it, it would take a long time in order for them to master the package, and I think the cost-benefit doesn't pay there… And there's huge scope for them making mistakes and errors, especially at this level of student... And the content of geometry at foundation and intermediate level just doesn't require that degree of investigation.

The second teacher was similarly sceptical as to whether working with DGS would directly benefit students (from his Year 9 upper set) in examination terms, but he saw it as having potential to increase their enjoyment and understanding:

> I hope that [working with DGS] makes them enjoy it more. I hope it gives them an understanding which you don't necessarily need to do the GCSE [examination] questions [for which] you just learn the fact and then you do it.

Teaching in an ICT suite, his normal practice was for students to work in pairs, constructing figures for themselves by following his step-by-step instructions at the interactive whiteboard. He regarded getting students to undertake such constructions as a worthwhile investment:

> I wanted them to get the idea of using it, because I want to be using it with them off and on over the next three years… I thought they made quite good progress on that and we didn't waste a lesson, as it were, learning. We actually learned some maths as well as how to use it.

The third teacher (working with upper sets in Years 7 and 8) had access to a set of laptops which enabled students to work in pairs, and her classroom was also equipped with an interactive whiteboard for work involving the whole class. She had carefully structured students' use of DGS to minimise technical complexities, typically requiring them only to drag prepared figures, but occasionally involving them in simple construction:

> It does add complications, because it's quite a difficult piece of software. So that's why we structure the work so they just have to move points. So they don't have to be complicated by that, they really can just focus on what's happening mathematically… There's very little actual construction that pupils do themselves. And although we do do some… it's very simple construction.

Teachers' thinking about the value of students learning to use DGS also appears to have been influenced by whether they took a pragmatic view of DGS as 'just a drawing program', or saw DGS construction as bringing out mathematical relations:

> One of the main parts of this lesson was that they could… see that the software works geometrically... And so when they were trying to measure the angle, that really brought out the idea of what is an angle… Just the action of doing it... and they really understood that angles, these three points that are on two lines, and what it means.

Teachers differed considerably, then, in the degree to which they involved students in carrying out DGS construction, manipulation and measurement. Decisions about involving students in these aspects of DGS use were shaped by teachers' assessment of the immediate demands and eventual benefits of such investment. These assessments, in turn, were influenced by whether teachers saw educative potential in the mathematically disciplined character of DGS use.

Working efficiently with geometric figures

All three teachers were concerned with efficiency in constructing and measuring the geometric figures used in classwork. The benchmark to which they referred was the familiar situation in which these processes were carried out by hand. In particular, they noted how, once a DGS figure had been constructed and the desired measurements specified, further examples could be created simply by dragging the figure, whereas to achieve this by hand required repetition of the whole drawing and measurement cycle. Teachers saw DGS as increasing the efficiency and accuracy with which figures could be created and measured, so expediting the pace and progress of lessons:

> Everything that we did there, I could have done by hand on the board, piece by piece by piece… It's very quick for me, I don't have to spend a long time drawing these things out. And then measuring the angles... We would make very little progress compared to what we've done already… It keeps the lesson moving at a good pace.

While use of DGS could remove difficulties that students experienced in drawing and measuring figures by hand, teachers also reported that some students experienced (analogous) difficulties in physically manipulating DGS, particularly by means of a touchpad or a defective mouse. Even with a well functioning mouse, problems remained:

> If they're supposed to click on a point, the mouse isn't quite on it, so they'll click and create a new point, and then when they move the point they are supposed to move, the angle doesn't change with it because they've attached it to a different point. So there's all sorts of little things that you constantly have to [attend to].

Accordingly, this teacher reported that he gave priority to instructing students in techniques for simplifying DGS figures through deleting spurious points and lines. The classes that we observed were relatively inexperienced in making use of DGS, but such difficulties appeared to be the norm, accentuated by the very occasional use of this technology.

Teachers were agreed, then, that using DGS could increase the efficiency with which geometric figures were created and measured, so expediting the pace and progress of lessons. At the same time, some students experienced difficulties in physically manipulating DGS tools not dissimilar to those associated with the use of ruler, compass, and protractor. This required close attention by the teacher, and the consequences could be alleviated by training students to identify and remove spurious material.

Managing apparent anomalies of measurement

All three teachers commented on how they managed apparent anomalies in the behaviour of DGS, where the measurements produced by the system diverged from what might be expected.

One such type of result arose in measuring reflex angles:

> Sometimes it doesn't do quite what you expect. For example, if you mark an angle… it will always mark the one less than 180 and that’s not always what you want it to do. And when you move things round sometimes the angle that it’s displaying isn't quite what you expected.

The first two teachers were careful to avoid exposing students to situations of this type. For example, in the ‘circle theorem’ lessons they only considered situations where the angle at the centre of the circle was obtuse, finally dragging it to a value of 180 degrees (so that its arms formed a diameter) in order to establish the final target result about the angle in a semi-circle. The issue of what might happen when this angle was dragged beyond that position to become reflex was not considered. Likewise, in a lesson on polygon angle sums, one of these teachers did inadvertently create a reflex angle, but quickly dragged it back once he realised what had happened. By contrast, in her lesson on this same topic, the third teacher made no move to prevent students encountering reflex angles, and indeed used this as a springboard for more extended mathematical discussion:

> For me, success is when the kids produce something and then say, 'This can't be right because it's not what I expect.'… So that happened in slightly different ways around the room, but it was one of the key things that the kids learned. That you can't assume that

> what you've got in front of you is actually what you want. And you have to look at it… and question it.

Another type of anomalous result could arise as a result of numeric values being rounded. For example, in the circle-theorem lessons, the first two teachers carefully set defaults and managed dragging so as to avoid students meeting situations which might obscure the underlying rule or impede them from finding it:

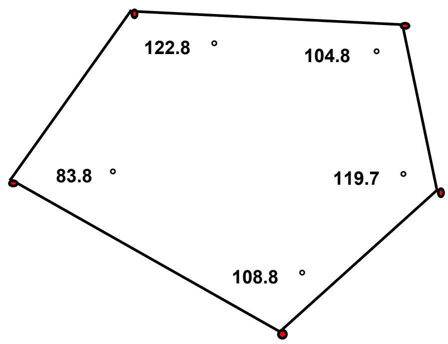
> I made sure the angles were always integer values... That way you don't have half angles to deal with. So if you noticed, the angle at the centre was always an even number of degrees because that way the angle at the outside can be halved quite successfully... So I did that to help make it a little bit easier for them to spot the rule.

Likewise, in the lessons on polygons, episodes occurred where the sum of angles diverged from the expected value. Again, it was the third teacher who used this anomaly to promote more extended discussion and mathematisation.

Teachers differed considerably, then, in the degree to which they sought to avoid exposing students to apparent anomalies in DGS operation. Such decisions were influenced by whether teachers saw such situations as providing opportunities for mathematisation, and for instilling a critical attitude to computer results.

Evidencing geometric properties through dragging figures

For all three teachers, manipulating figures through dragging was the most important feature of DGS use.



In the *polygon angle-sum lessons,* the DGS figures employed took the form of simple polygons with the measures of all angles marked (see adjacent figure). Dragging points was treated as a means of generating different examples of each type of polygon. The development of these lessons followed an inductive sequence, starting with the familiar cases of the triangle and quadrilateral, using these to introduce the dragging approach; then proceeding to pentagon and later polygons, so establishing a table of angle sums from which a pattern could be formulated.

One teacher suggested that arbitrarily halting the dragging conveyed a sense of selection from amongst many figures:

> The fact they can see it changing as you're dragging and dropping it, makes the difference. It's a bit more convincing for them. And then also at one stage I got one of them to actually tell me where to stop… so it wasn't always me that was choosing it.

Indeed, in his lesson, this teacher explicitly introduced the idea of choosing at random:

> We've just picked four triangles at random and shown that that's true. And there's no way that could have happened by accident.

As well as calculating angle sums, teachers wanted to show that any polygon could be decomposed into triangles. One teacher drew by hand onto each projected DGS figure to show a triangular decomposition. Another commented that, had time permitted in her lesson, she would have asked students to add segments to their DGS figures in order to triangulate them. In both these approaches to triangulating the polygon, as well as in the classroom discussion that we observed, the rationale for doing so tended to be taken for granted: it could have been made more explicit that the purpose was to decompose the angles of the polygon into triangular sets.

Although teachers described the aims of their *circle theorem lessons* in terms which organised results deductively, the way in which they expressed the angle-at-the-circumference property in dynamic terms was notable:

> The objectives were... to learn that the angle at the outside was [half] the angle in the middle, and also that it therefore didn't change as it moved around the circumference.

In fact, the dynamically striking result about the moving angle-at-the-circumference was actually presented first in lessons. Dragging was used to convey a sense of the unchanging measure of the moving angle:

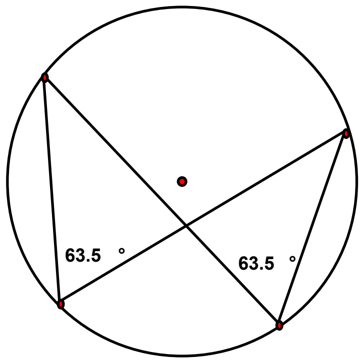
> The technology helps because they can actually see it getting dragged round, they see the angle doesn't change and they are much more convinced.

One teacher noted how this could provoke students into making sense of what was going on:

> I heard one of the boys, for example, saying 'There's something wrong with this, it's always the same angle wherever I move it to'. And then it dawned on him that that was the whole point!

By contrast, treatment of the other results appealed to dragging more as a means of generating different examples (and so a data pattern), or of examining a special case.

Although teachers did not comment on this, they were observed to incorporate episodes into their lessons in which the figure was elaborated so that the 'dynamic' image of the moving angle-at-the-circumference was (tacitly) related to the more customary 'static' image of two fixed angles-at-the-circumference (see adjacent figure). Intentionally or not, these episodes can be seen as serving to establish an important relationship between the dynamic figures employed in these lessons, and the static figures which students would encounter once they moved on to tackle exercises on the page.



To summarise, then, dragging of figures was employed to evidence properties in two ways. Most commonly, it was used to *examine multiple examples or special* *cases* of a geometric figure, without particular attention to variation during the dragging process itself, other than in evoking the multiplicity of possibilities. More occasionally, dragging was used to *examine dynamic variation* (notably non-variation) in a geometric figure during the dragging process, and this could extend to demarcating the domain over which a property held. Most strikingly, regardless of the type of dragging employed, *consideration of geometric properties was almost always focused on the effects of dragging on numeric measures*.

Concluding discussion

These approaches to using dynamic geometry to teach about angle properties have been developed by teachers in well-regarded departments, motivated by the potential they saw to enhance existing practice. Teachers were able both to demonstrate this enhanced practice in the lessons observed, and to articulate key features in subsequent interviews. We hope that identifying central issues and describing (sometimes differing) strategies for addressing them will be of interest and help to other teachers incorporating dynamic geometry into their practice.

Naturally, these approaches reflect the emphases of current secondary mathematics curriculum and assessment. The focus on angle measures matches the prominence of 'angle-chasing' problems in today's texts and tests. The accent on dragging figures in order to generate sets of measures fits the prevalence of data pattern generalisation. The tight structuring of investigative tasks addresses current concerns for itemised knowledge in standard form. Nevertheless, the practice of the third teacher in allowing students to encounter apparent anomalies in the operation of computational tools and viewing these as opportunities for mathematisation also addresses an important aspect of 'using and applying mathematics' which is currently not directly assessed.

Equally, these approaches are finely tuned to divergences between the way in which properties are expressed in dynamic as opposed to static figures. For example, care was taken to establish continuity between dynamic computer-based and static paper-based representations of the angles-at-the-circumference property. This constitutes the main mathematical enhancement that such approaches seem to offer to students' understanding of angle properties: adding a dynamic perspective –and coordinating it with the static perspective– to build a richer conceptual system.

However, looking beyond the current curriculum, from a broader mathematical perspective, the emphasis on mediating geometric properties through numeric measures, and on data pattern generalisation, could be seen as rather narrow. This is certainly a concern of recent reports and recommendations on the teaching of geometry. Hence, a future article (Ruthven, 200#) will suggest how the approaches observed in use here might be adapted and extended so as to support classroom activity involving a broader range of mathematical thinking, expanding –in particular– its visuo-spatial and logico-deductive aspects.

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